

# CKM angles from non-leptonic B decays using SU(3) flavour symmetry

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**Abstract.** We discuss the determination of the CKM angles  $\gamma$  and  $\alpha$  using recent data from non-leptonic B decays together with flavour symmetries. Penguin effects are controlled by means of the CP-averaged branching ratio  $B_d \rightarrow \pi^\pm K^\mp$ . The information from  $\mathcal{A}_{CP}(B_d \rightarrow J/\Psi K_S)$  (two solutions for  $\phi_d$ ),  $R_b$  and  $\gamma$  allow us to determine  $\beta$ , even in presence of New Physics not affecting  $\Delta B = 1$  amplitudes. In this context we address the question of to what extent there is still space for New Physics.

**PACS.** 13.25Hw Hadronic decays of mesons – 11.30Er CP violation

## 1 Introduction

B physics is one of the most fertile testing grounds to check the CKM mechanism of CP violation in the SM [1], but also to look for the first signals of New Physics [2] in the pre-LHC era.

The huge effort at the experimental level at the B factories and future hadronic machines [3] has produced, already, several impressive results. First, the measurement of  $\sin \phi_d$  from the mixing induced CP asymmetry of the decay  $B_d \rightarrow J/\Psi K_S$ . Second, the measurement of a series of non-leptonic B decays:  $B_d \rightarrow \pi K$ ,  $B_d \rightarrow \pi\pi$  and in the future hadronic machines  $B_s \rightarrow KK$  will be also accessible.

These non-leptonic B decays play a fundamental role in the determination of the CKM angle  $\gamma$ . The main problem in analysing them is how to deal with hadronic matrix elements and how to control penguin contributions. Our approach [4,5,6,7] extract the maximal possible information from data using flavour symmetries to try to reduce as much as possible the uncertainties associated to QCD hypothesis.

## 2 CKM angle $\gamma$ from non-leptonic decays:

$B_d \rightarrow \pi\pi$ ,  $B_d \rightarrow \pi K$  and  $B_s \rightarrow KK$

We start writing down a general amplitude parametrization of  $B_d \rightarrow \pi^+\pi^-$  in the SM [4,6]:

$$A(B_d^0 \rightarrow \pi^+\pi^-) = \mathcal{C} (e^{i\gamma} - d e^{i\theta})$$

All the hadronic information is collected in

$$d e^{i\theta} \equiv \frac{1}{R_b} \left( \frac{A_{\text{pen}}^{ct}}{A_{\text{CC}}^u + A_{\text{pen}}^{ut}} \right) \quad \mathcal{C} \equiv \lambda^3 A R_b (A_{\text{CC}}^u + A_{\text{pen}}^{ut})$$

where  $A_{\text{CC}}^u$  are current-current contributions and  $A_{\text{pen}}^{qt}$  are differences between penguin contributions with a quark  $q = u, c$  and a quark top inside the loop.

This amplitude allow us to construct the corresponding CP asymmetries [4,6]:

$$\mathcal{A}_{\text{CP}}^{\text{dir}} = \text{func}(d, \theta, \gamma) \quad \mathcal{A}_{\text{CP}}^{\text{mix}} = \text{func}(d, \theta, \gamma, \phi_d)$$

Following a similar procedure we can write down the amplitude for a closely related process:

$$A(B_s^0 \rightarrow K^+K^-) = \left( \frac{\lambda}{1 - \lambda^2/2} \right) \mathcal{C}' \left[ e^{i\gamma} + \left( \frac{1 - \lambda^2}{\lambda^2} \right) d' e^{i\theta'} \right]$$

whose corresponding asymmetries will depend on [4,6]:

$$\mathcal{A}_{\text{CP}}^{\text{dir}} = \text{func}(d', \theta', \gamma) \quad \mathcal{A}_{\text{CP}}^{\text{mix}} = \text{func}(d', \theta', \gamma, \phi_s)$$

The crucial point, here, is that the hadronic parameters  $d'$ ,  $\theta'$  and  $\mathcal{C}'$ , has exactly the same functional dependence on the penguins that  $d$ ,  $\theta$  and  $\mathcal{C}$ , except for the interchange of a  $d$  quark by an  $s$  quark.

As a consequence, both processes can be related via U-spin symmetry, reducing the total number of parameters to five:  $\gamma$ ,  $d$ ,  $\theta$ ,  $\phi_d$  and  $\phi_s$ . At this point, one must check the sensitivity of the results to the breaking of U-spin symmetry. This is explained in subsection 2.2.

Looking a bit more in detail, one finds that  $d$  is indeed not a free parameter, but it can be constrained or substituted using an observable called  $H$  [7,6]:

$$H \equiv \frac{1}{\epsilon} \left| \frac{\mathcal{C}'}{\mathcal{C}} \right|^2 \left[ \frac{M_{B_d}}{M_{B_s}} \frac{\Phi(\frac{M_K}{M_{B_s}}, \frac{M_K}{M_{B_s}})}{\Phi(\frac{M_\pi}{M_{B_d}}, \frac{M_\pi}{M_{B_d}})} \frac{\tau_{B_s}}{\tau_{B_d}} \right] \left[ \frac{\text{BR}(B_d \rightarrow \pi^+\pi^-)}{\text{BR}(B_s \rightarrow K^+K^-)} \right]$$

This quantity requires the knowledge of  $\text{BR}(B_s \rightarrow K^+K^-)$ , which is still not available. However, we can already now

evaluate  $H$  by making contact with the B factories and substitute  $B_s \rightarrow K^+ K^-$  by  $B_d \rightarrow \pi^\pm K^\mp$ . These two processes differ by the spectator quark and certain exchange and penguin annihilation topologies that are expected to be small [8]. This leads to the following value for  $H$  [9]:

$$H \approx \frac{1}{\epsilon} \left( \frac{f_K}{f_\pi} \right)^2 \left[ \frac{\text{BR}(B_d \rightarrow \pi^+ \pi^-)}{\text{BR}(B_d \rightarrow \pi^\mp K^\pm)} \right] = 7.5 \pm 0.9 \quad (1)$$

Due to the dependence of  $H$  only on  $\cos\theta \cos\gamma$  in the U-spin limit, we obtain immediately a constrained range for  $d$ :  $0.2 \leq d \leq 1$ . Also, using the exact expression for  $H$  we can obtain  $d$  as a function of  $H$ ,  $\theta$  and  $\gamma$ .

It is important to insist here that once the data on the branching ratio of  $B_s \rightarrow KK$  will be available, the spectator quark hypothesis will not be necessary and only U-spin breaking effects will be important.

## 2.1 Prediction for CKM-angle $\gamma$

Let's take as starting point the general expression [6]:

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+ \pi^-) = \mp \left[ \frac{\sqrt{4d^2 - (u + vd^2)^2} \sin \gamma}{(1 - u \cos \gamma) + (1 - v \cos \gamma)d^2} \right] \quad (2)$$

where  $u, v, d = F_i(\mathcal{A}_{\text{CP}}^{\text{mix}}, H, \gamma, \phi_d(B_d \rightarrow J/\Psi K_s); \xi, \Delta\theta)$ . The parameters  $\xi, \Delta\theta$  will account for the U-spin breaking and are discussed in subsection 2.2.

Using present world average for  $\sin\phi_d = 0.734 \pm 0.054$ , one obtains two possible solutions for the weak mixing angle:

$$\phi_d = (47_{-4}^{+5})^\circ \vee (133_{-5}^{+4})^\circ.$$

We will refer later on to these two solutions like scenario A and B, respectively.

Concerning experimental data, the situation is still uncertain, but improving. Present naive average of Belle and Babar data is [10]:

$$\begin{aligned} \mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+ \pi^-) &= -0.38 \pm 0.16 \\ \mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+ \pi^-) &= +0.58 \pm 0.20 \end{aligned}$$

The intersection of the two experimental ranges of  $\mathcal{A}_{\text{CP}}^{\text{dir}}$  and  $\mathcal{A}_{\text{CP}}^{\text{mix}}$  allow us, using Eq. (2), to determine the range for  $\gamma$ . The first range, corresponding to take  $\phi_d = 47^\circ$  is:

$$32^\circ \lesssim \gamma \lesssim 75^\circ \quad (3)$$

For the second solution  $\phi_d = 133^\circ$  one obtains:

$$105^\circ \lesssim \gamma \lesssim 148^\circ \quad (4)$$

Both plots are symmetric (see [6,11]). This is a consequence of the symmetry  $\phi_d \rightarrow 180^\circ - \phi_d$ ,  $\gamma \rightarrow 180^\circ - \gamma$  that Eq. (2) exhibits. It is remarkable the stability of the range for  $\gamma$  if we compared it with previous analysis [11].

## 2.2 Sensitivity to parameters $H$ , $\xi$ and $\Delta\theta$

Here we will analyze the sensitivity of the determination of  $\gamma$  on the variation of the different hadronic parameters.

### 2.2.1 $H$ and the spectator quark hypothesis

Let's fix the solution  $\phi_d = 47^\circ$  and take the experimental branching ratios of  $B_d \rightarrow \pi\pi$  and  $B_d \rightarrow \pi K$  to determine  $H$ . We vary  $H$  inside its experimental range Eq. (1) at one, two and three sigmas to take into account the uncertainty associated to the spectator quark hypothesis. We find at one sigma a very mild influence in the determination of  $\gamma$ . The error induced in the range of  $\gamma$  is about  $\pm 2^\circ$ .

For the very conservative range of up to three sigmas we find a maximal error of  $6^\circ$ . Moreover, if the experimental value of  $H$  tends to increase the range for  $\gamma$  tends to decrease, allowing for a narrower determination.

Finally, the uncertainty associated to  $H$  will be drastically reduced once the  $\text{BR}(B_s \rightarrow KK)$  is known and  $H$  will be taken safely in a narrower range.

### 2.2.2 U-spin breaking: $\xi$ and $\Delta\theta$

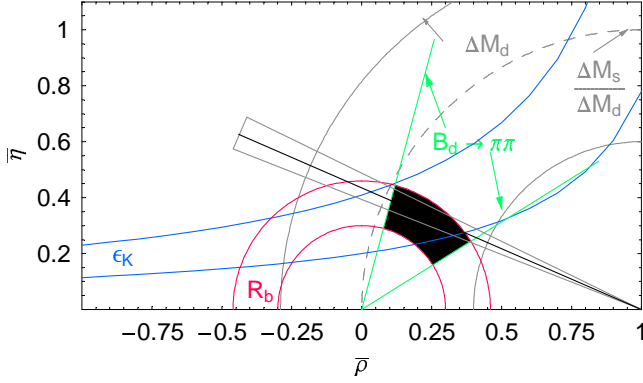
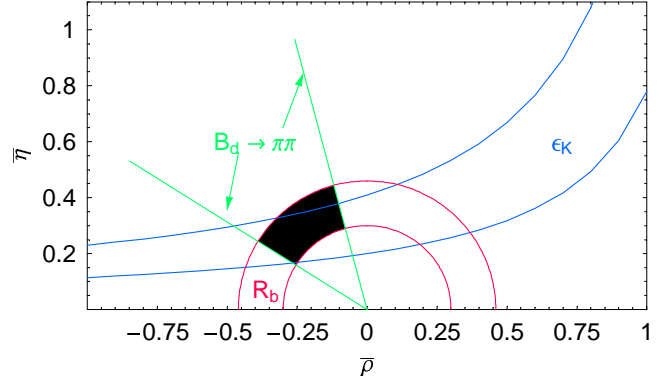
U-spin breaking is the most important uncertainty. We will follow two different strategies to keep it under control:

- Once the data from the CP asymmetries and branching ratio of  $B_s \rightarrow KK$  will be available and  $\phi_s$  will be measured from the CP-asymmetry of  $B_s \rightarrow J/\Psi\phi$ , we will be able to *test* directly from data U-spin breaking. Taking  $\phi_d$  from  $B_d \rightarrow J/\Psi K_S$  we will have 4 observables (the CP asymmetries) and 3 unknowns ( $d, \theta, \gamma$ ). Then, we can add  $d'$  as another free parameter and data will tell us the amount of U-spin breaking.
- Already now, we can define two quantities  $\xi = d'/d$  and  $\Delta\theta = \theta' - \theta$  that parametrizes the amount of U-spin breaking. In order to test the sensitivity of  $\gamma$  to the variation of these parameters, we allow them to vary in a range. If we allow for a very large variation of  $\xi$  between 0.8 and 1.2, the larger error in the determination of  $\gamma$  is of  $\pm 5^\circ$ . Concerning  $\Delta\theta$ , its influence is negligibly small, a variation of  $40^\circ$  induces an error of at most 1 degree.

Other studies on U-spin breaking can be found in [12].

## 3 Determination of CKM angles $\alpha$ and $\beta$ in SM and with New Physics in the mixing

Next point is how to determine  $\alpha$  and  $\beta$  [9]. Here, in addition, we will also allow for Generic New Physics affecting the  $B_d^0 - \bar{B}_d^0$  mixing, but not to the  $\Delta(B, S) = 1$  decay amplitudes, i.e, this type of New Physics is consistent with the determination of  $\gamma$  explained in the previous section. Our inputs are [9,13]:

Fig. 1.  $\phi_d = 47^\circ$ . SCENARIO AFig. 2.  $\phi_d = 133^\circ$ . SCENARIO B

- $R_b \equiv \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right|$  obtained from exclusive/inclusive transitions mediated by  $b \rightarrow u\ell\bar{\nu}_\ell$  and  $b \rightarrow c\ell\bar{\nu}_\ell$ . Two important remarks are: a) This is an observable practically insensitive to New Physics, b) from  $R_b^{\max} = 0.46$  we can extract a robust maximum possible value for  $\beta$ :  $|\beta|_{\max} = 27^\circ$ , respected by the two scenarios.
- $\gamma$  obtained as discussed in previous sections.
- $\phi_d$  from  $\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \rightarrow J/\psi K_S)$  is used as an input for the CP asymmetries of  $B_d \rightarrow \pi\pi$ , but NOT to determine  $\beta$ , since we assume that New Physics could be present. Also  $\Delta M_d$  and  $\Delta M_s/\Delta M_d$  are not used as inputs, due to their sensitivity to New Physics.

Using these inputs we obtain two possible determinations for  $\alpha$ ,  $\beta$  and  $\gamma$ , corresponding to the two possible values of  $\phi_d$ .

### 3.1 Scenario A: Compatible with SM

This scenario corresponds to the first solution  $\phi_d = 47^\circ$ , which implies the range for  $\gamma$  given in Eq. (3). Together with  $R_b$  we obtain the black region shown in Fig 1. It implies the following prediction for the CKM angles:

$$78^\circ \leq \alpha \leq 136^\circ \quad 13^\circ \leq \beta \leq 27^\circ \quad 32^\circ \leq \gamma \leq 75^\circ$$

and the error associated with  $\xi \in [0.8, 1.2]$  is  $\Delta\alpha = \pm 4^\circ$ ,  $\Delta\beta = \pm 1^\circ$  and  $\Delta\gamma = \pm 5^\circ$ . It is interesting to notice that this region is in good agreement with the usual CKM fits [14]. To illustrate it we have shown in Fig. 1 also the prediction from the SM interpretation of different observables:  $\Delta M_d$ ,  $\Delta M_s/\Delta M_d$ ,  $\epsilon_K$  and  $\phi_d^{\text{SM}} = 2\beta$ .

### 3.2 Scenario B: New Physics

The second solution:  $\phi_d = 133^\circ$  *cannot* be explained in the SM context and requires New Physics contributing to the mixing[9,13]. Models with New sources of Flavour mixing can account for this second solution with only two very general requirements [9]: a) The effective scale of New Physics is larger than the electroweak scale and b) the adimensional effective coupling ruling  $\Delta B = 2$  processes can

always be expressed as the square of two  $\Delta B = 1$  effective couplings. Supersymmetry provides a perfect example, in particular, through the contribution of gluino mediated box diagrams with a mass insertion  $\delta_{bL}^D d_L$ [9].

In this case,  $\gamma$  lies in the second quadrant Eq. (4) and  $\beta$  is indeed smaller than in the previous scenario. The result is still consistent with the  $\epsilon_K$  hyperbola.  $\Delta M_{d,s}$  are not shown here, since they would be affected by New Physics. The black region obtained (see Fig.2) corresponds to the following prediction for the CKM angles:

$$22^\circ \leq \alpha \leq 60^\circ \quad 8^\circ \leq \beta \leq 22^\circ \quad 105^\circ \leq \gamma \leq 148^\circ$$

with same errors associated to  $\xi$  as in Scenario A. It is interesting to remark that this second solution has also interesting implications for certain rare decays like  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ [9,15]. Using this second solution we find a better agreement with experiment than with the SM solution. Concerning  $B_d \rightarrow \mu^+ \mu^-$ , we find also sizeable differences depending on the scenario used.

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